

## SECTION 2.2 SUMMARY

In general, polynomials can be graphed in one of two ways:

Use graph-shifting techniques with power functions.

General polynomial function.

1. Identify intercepts.

2. Determine each real zero and its multiplicity, and ascertain whether the graph crosses or touches the  $x$ -axis there.

3.  $x$ -intercepts (real zeros) divide the  $x$ -axis into intervals.

Test points in the intervals to determine whether the graph is above or below the  $x$ -axis.

4. Determine the end behavior by investigating the end behavior of the highest degree monomial.

5. Sketch the graph with a smooth curve.

## SECTION 2.2 EXERCISES

### SKILLS

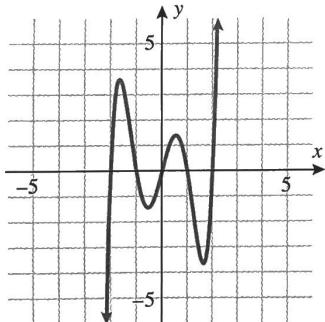
**In Exercises 1–10, determine which functions are polynomials, and for those that are, state their degree.**

1.  $g(x) = (x + 2)^3(x - \frac{3}{5})^2$
2.  $g(x) = (x - \frac{1}{4})^4(x + \sqrt{7})^2$
3.  $g(x) = x^5(x + 2)(x - 6.4)$
4.  $g(x) = x^4(x - 1)^2(x + 2.5)^3$
5.  $h(x) = \sqrt{x} + 1$
6.  $h(x) = (x - 1)^{1/2} + 5x$
7.  $F(x) = x^{1/3} + 7x^2 - 2$
8.  $F(x) = 3x^2 + 7x - \frac{2}{3x}$
9.  $G(x) = \frac{x + 1}{x^2}$
10.  $H(x) = \frac{x^2 + 1}{2}$

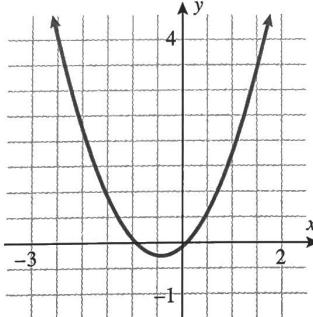
**In Exercises 11–18, match the polynomial function with its graph.**

- |                        |                           |                          |                                |
|------------------------|---------------------------|--------------------------|--------------------------------|
| 11. $f(x) = -3x + 1$   | 12. $f(x) = -3x^2 - x$    | 13. $f(x) = x^2 + x$     | 14. $f(x) = -2x^3 + 4x^2 - 6x$ |
| 15. $f(x) = x^3 - x^2$ | 16. $f(x) = 2x^4 - 18x^2$ | 17. $f(x) = -x^4 + 5x^3$ | 18. $f(x) = x^5 - 5x^3 + 4x$   |

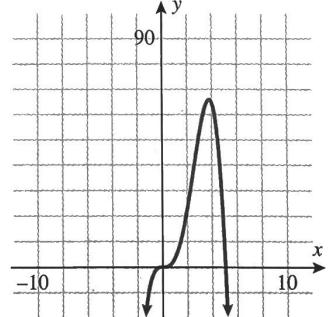
a.



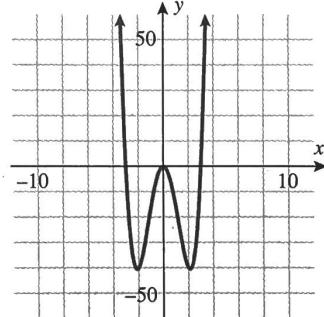
b.



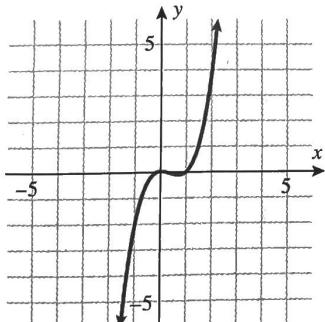
c.



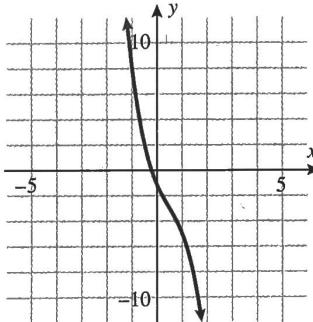
d.



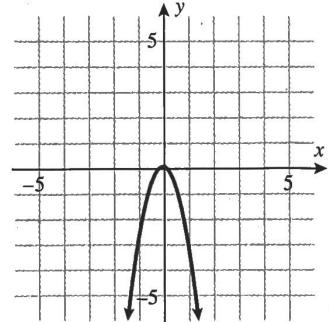
e.



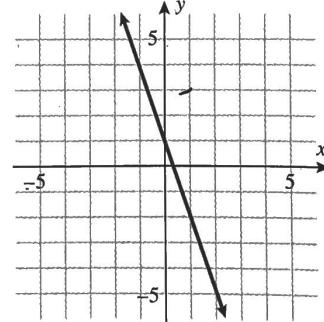
f.



g.



h.



In Exercises 19–24, graph each function by transforming a power function  $y = x^n$ .

19.  $f(x) = (x - 2)^4$

20.  $f(x) = (x + 2)^5$

21.  $f(x) = x^5 + 3$

22.  $f(x) = -x^4 - 3$

23.  $f(x) = 3 - (x + 1)^4$

24.  $f(x) = (x - 3)^5 - 2$

In Exercises 25–36, find all the real zeros (and state their multiplicities) of each polynomial function.

25.  $f(x) = 2(x - 3)(x + 4)^3$

26.  $f(x) = -3(x + 2)^3(x - 1)^2$

27.  $f(x) = 4x^2(x - 7)^2(x + 4)$

28.  $f(x) = 5x^3(x + 1)^4(x - 6)$

29.  $f(x) = 4x^2(x - 1)^2(x^2 + 4)$

30.  $f(x) = 4x^2(x^2 - 1)(x^2 + 9)$

31.  $f(x) = 8x^3 + 6x^2 - 27x$

32.  $f(x) = 2x^4 + 5x^3 - 3x^2$

33.  $f(x) = -2.7x^3 - 8.1x^2$

34.  $f(x) = 1.2x^6 - 4.6x^4$

35.  $f(x) = \frac{1}{3}x^6 + \frac{2}{5}x^4$

36.  $f(x) = \frac{2}{7}x^5 - \frac{3}{4}x^4 + \frac{1}{2}x^3$

In Exercises 37–50, find a polynomial (there are many) of minimum degree that has the given zeros.

37.  $-3, 0, 1, 2$

38.  $-2, 0, 2$

39.  $-5, -3, 0, 2, 6$

40.  $0, 1, 3, 5, 10$

41.  $-\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

42.  $-\frac{3}{4}, -\frac{1}{3}, 0, \frac{1}{2}$

43.  $1 - \sqrt{2}, 1 + \sqrt{2}$

44.  $1 - \sqrt{3}, 1 + \sqrt{3}$

45.  $-2$  (multiplicity 3),  $0$  (multiplicity 2)

46.  $-4$  (multiplicity 2),  $5$  (multiplicity 3)

47.  $-3$  (multiplicity 2),  $7$  (multiplicity 5)

48.  $0$  (multiplicity 1),  $10$  (multiplicity 3)

49.  $-\sqrt{3}$  (multiplicity 2),  $-1$  (multiplicity 1),  $0$  (multiplicity 2),  $\sqrt{3}$  (multiplicity 2)

50.  $-\sqrt{5}$  (multiplicity 2),  $0$  (multiplicity 1),  $1$  (multiplicity 2),  $\sqrt{5}$  (multiplicity 2)

In Exercises 51–68, for each polynomial function given: (a) list each real zero and its multiplicity; (b) determine whether the graph touches or crosses at each  $x$ -intercept; (c) find the  $y$ -intercept and a few points on the graph; (d) determine the end behavior; and (e) sketch the graph.

51.  $f(x) = (x - 2)^3$

52.  $f(x) = -(x + 3)^3$

53.  $f(x) = x^3 - 9x$

54.  $f(x) = -x^3 + 4x^2$

55.  $f(x) = -x^3 + x^2 + 2x$

56.  $f(x) = x^3 - 6x^2 + 9x$

57.  $f(x) = -x^4 - 3x^3$

58.  $f(x) = x^5 - x^3$

59.  $f(x) = 12x^6 - 36x^5 - 48x^4$

60.  $f(x) = 7x^5 - 14x^4 - 21x^3$

61.  $f(x) = 2x^5 - 6x^4 - 8x^3$

62.  $f(x) = -5x^4 + 10x^3 - 5x^2$

63.  $f(x) = x^3 - x^2 - 4x + 4$

64.  $f(x) = x^3 - x^2 - x + 1$

65.  $f(x) = -(x + 2)^2(x - 1)^2$

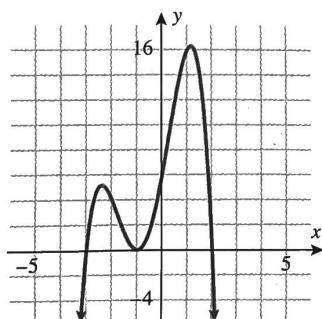
66.  $f(x) = (x - 2)^3(x + 1)^3$

67.  $f(x) = x^2(x - 2)^3(x + 3)^2$

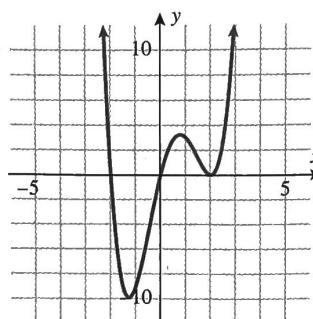
68.  $f(x) = -x^3(x - 4)^2(x + 2)^2$

In Exercises 69–72, for each graph given: (a) list each real zero and its smallest possible multiplicity; (b) determine whether the degree of the polynomial is even or odd; (c) determine whether the leading coefficient of the polynomial is positive or negative; (d) find the  $y$ -intercept; and (e) write an equation for the polynomial function (assume the least degree possible).

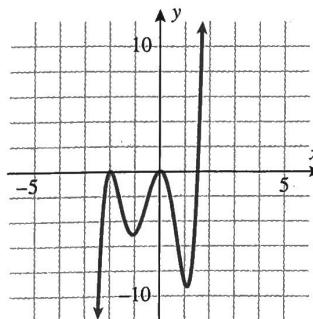
69.



70.



71.



72.

